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KD-6221

B. E. - II (IC) (Sem. III) Examination

December - 2012

Engineering Mathematics - Paper - III

(OLD)

Time : **3** Hours] [Total Marks : **100**

Instructions : (1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. E. - II (IC) (Sem. III)**

Name of the Subject :

☛ **Engineering Mathematics - Paper - III (Old)**

☛ Subject Code No. : **6 2 2 1** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Figures to the right indicate full marks.

1 (a) Answer the following. **10**

(1) Find grad ϕ when ϕ is given by $\phi = x^3 + y^3 + 3xyz$ at the point (1, 2, −1).

(2) Find curl $\vec{V}$ where $\vec{V} = xyz\,\hat{i} + 3x^2y\,\hat{j} + \left(xz^2 - y^2z\right)\hat{k}$

(3) Evaluate the integral

$$\int_0^a \int_0^a \int_0^a (yz + zx + xy)\,dx\,dy\,dz$$

(4) Write the Fourier coefficients for the function f(x) defined in the interval $(0, 2\pi)$.

(5) What are odd functions ? Give the Euler's formulae for the odd function defined in $(-\pi, \pi)$.

(b) Attempt any four. **12**

(1) Evaluate $\int_0^1 \int_0^x e^{y/x}\,dy\,dx$.

(2) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\,dy\,dx$ by changing the order of integration.

(3) Change into polar co-ordinates and evaluate $\int_0^{\infty}\int_0^{\infty} e^{-\left(x^2+y^2\right)}dydx$.

(4) Find, by double integration, the volume generated by revolving the cardioid $r=a(1+\cos\theta)$ about the initial line.

(5) Find the mass of a plate in the shape of the curve $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$, the density being given by $\rho=bxy$.

2 (a) Attempt any two. **6**

(1) A particle moves on the curve $x=2t^2, y=t^2-4t, z=3t-5$ where t is the time. Find the components of velocity and acceleration at time t=1 in the direction $\hat{i}-3\hat{j}+2\hat{k}$.

(2) Find the directional derivative of the function $f=xy^2+yz^3$ at the point (2, -1, 1) in the direction of the vector $\hat{i}+2\hat{j}+2\hat{k}$.

(3) Define the solenoidal vector. Show that the vector $\vec{V}=(x+3y)\hat{i}+(y-3z)\hat{j}+(x-2z)\hat{K}$ is solenoidal.

(b) Attempt any two. **8**

(1) If $\vec{f}(t)=t\hat{i}+\left(t^2-2t\right)\hat{j}+\left(3t^2+3t^3\right)\hat{k}$ find $\int_0^1 \vec{f}(t)dt$.

(2) Verify divergence theorem for $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ taken over the region bounded by the cylinder $x^2+y^2=4$, $z=0$ and $z=3$.

(3) Apply Green's theorem to evaluate $\oint_c[(y-\sin x)dx+\cos xdy]$ where C is the plane triangle enclosed by the lines $y=0, x=\frac{\pi}{2}$ and $y=\frac{2}{\pi}x$.

3 (a) Expand f(x)=x as a half range sine series in 0<x<2. **4**

(b) Attempt any two. **10**

(1) Obtain the Fourier series for $f(x)=e^{-x}$ in the interval $0<x<2\pi$.

(2) Find the Fourier expansion for the function

$f(x) = x - x^2, \ -1 < x < 1$.

(3) Find the Fourier series to represent the function

$$f(x) = \begin{cases} -k, \ when \ -\pi < x < 0 \\ k, \ when \ 0 < x < \pi \end{cases}$$

Deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots$.

4 (a) Do as directed. **10**

(1) Define Beta function and evaluate $\beta(2,3)$.

(2) Find the value of $erf(\infty)$.

(3) Obtain Laplace transform of $f(t) = t^n$.

(4) Write one dimensional wave equation and give physically consistent solution to the same equaiton.

(5) Write Cachy-Riemann equations in polar form.

(b) Attempt any two. **6**

(1) Show that

$$\beta(p,q) = \int_0^\infty \frac{y^{q-1}}{(1+y)^{p+q}} dy$$

(2) Prove that

$$\int_0^2 \left(8 - x^3\right)^{-1/3} dx = \frac{1}{3} \Gamma\left(1/3\right) \Gamma\left(2/3\right)$$

(3) Show that

$$\int_0^\infty e^{-x^2 - 2ax} dx = \frac{\sqrt{\pi}}{2} e^{a^2} \left(1 - erf(a)\right)$$

(c) Solve any two. **6**

(1) $(mz - ny)p + (nx - lz)q = ly - mx$

(2) $\frac{y^2 z}{x} p + xzq = y^2$

(3) $\left(x^2 - y^2 - z^2\right) p + 2xyq = 2xz$

5 (a) Attempt any one. **6**

(1) An insulated rod of length l has its ends A and B maintained at 0^0C. and 100^0C respectively until steady state conditions prevail. If B is suddenly reduced to 0^0C and maintained at 0^0C, find the temperature at a distance x from A at time t.

(2) Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ which satisfies the conditions

$u(o,y)=u(l,y)=u(x,o)=0$ and $u(x,a)=\sin\frac{n\pi x}{l}$.

(b) Attempt any two. 8

(1) Find the Laplace transofrm of f(t) defined as

$$f(t)=\begin{cases} t & , \ o<t<4 \\ 5, & , \ t>4 \end{cases}.$$

(2) Find the inverse Laplace transform of $\frac{4S+15}{16S^2-25}$

(3) Solve the equation

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x=e^{-t}\sin t, \quad x(0)=0, \quad x'(0)=1.$$

6 (a) Attempt any two. 8

(1) Determine the analytic function w=u+iv whose real part is cos x coshy.

(2) Determine the region of the w-plane into which the region $\frac{1}{2}\le x\le 1$ and $\frac{1}{2}\le y\le 1$ is mapped by the transformation $w=z^2$.

(3) Find the bilinear transformation which maps the points $z=1,-i,-1$ into the points $w=i,o,-i$.

(b) Attempt any two. **6**

(1) Evaluate $\int_0^{3+i} z^2dz$, along the real axis to 3 and then vertically to 3+i.

(2) Evaluate $\oint_c \frac{z+4}{z^2+2z+5}dz$, where C is the circle $|z|=1$.

(3) Evaluate $\oint_c \frac{e^{-2z}}{(z+1)^3}dz$, where C is the circle $|z|=2$.